PERGAMON

Intl. Trans. in Op. Res. 7 (2000) 51–65

INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

www.elsevier.com/locate/orms

Decision criteria with partial information

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Abstract

In this paper we use an extreme point approach to analyze some usual decision criteria for multiple attribute decision-making problems when partial information about the importance of the attributes is available. The obtained results show that the decision criteria to be chosen depend not only on the rationality principles, but also on the structure of the information set. We apply the obtained criteria to problems where the set of actions to be evaluated are either in qualitative and/or quantitative scales.

Keywords: Decision; Information; Criteria

1. Introduction

The objective of this paper is to extend the use of some usual decision criteria for multi-attribute decision making when partial information about the importance of the attributes is available. Information about the importance of the attributes can be specified in different ways and with different levels of precision. A natural way to relax the task of determining the weighting coefficients is to establish interval criterion weights. Steuer (1976) and Mármol et al. (1998a,b) have provided procedures to obtain useful representations of these sets. Another way to supply information is to state linear relations among weights which can be seen as intercriteria preference (Bana e Costa, 1990; Carrizosa et al., 1995; Kirkwood and Sarin, 1985).

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PII: S0969-6016(99)00026-X

Particular cases are those which are applied in Data Envelopment Analysis (see Cook and Kress, 1994, 1996).

In general, we consider n actions or alternatives $\{A_i\}$, $i = 1, \ldots, n$ (projects, products, etc.) to be evaluated with respect to k attributes $\{C_j\}$, $j = 1, \ldots, k$. Let a_{ij} be the value, worth or utility of alternative A_i with respect to attribute C_j. Originally, the D-M would evaluate each alternative by

$$v(A_i) = (a_{i1}, \ldots, a_{ik})$$

and in order to compare them a criterion must be applied.

We only consider those criteria which are compatible with Pareto dominance. Therefore, when no information is available the D-M accepts that A_i is better than A_j if $v(A_i) \gneqq v(A_j)$. This relationship in mathematical terms is:

$$v(A_i)e_l \geq v(A_j)e_l \quad \forall l = 1, \ldots, k, \tag{1}$$

with at least one strict inequality and where e_l is the vector with 1 in the l-th entry and zero everywhere else. These sets of inequalities are equivalent to

$$v(A_i)\lambda \geq v(A_j)\lambda \quad \forall \lambda \in \Lambda^{\geq}, \tag{2}$$

with at least one strict inequality and where

$$\Lambda^{\geq} = \left\{ \lambda \in \mathbb{R}^k : \sum_{i=1}^{k} \lambda_i = 1, \lambda_i \geq 0 \right\}.$$

Notice that λ_i can be seen as the importance or weight factor assigned to the i-th attribute and $\Lambda^{\geq}$ is the whole set of weights which are admissible in the decision process.

When some information on the importance of the attributes is available the sense of domination changes. Assume that we are given a closed, convex set $\Lambda \subset \Lambda^{\geq}$ of weights which defines all the admissible weights of the attributes. It should be noted that these types of representation of the information, are not exhaustive. Nevertheless, although they are not exhaustive they cover an important range of representations of partial information as can be seen by their use in many papers (see Kirkwood and Sarin, 1985; Salo and Hämäläinen, 1992; Weber, 1987, among others). Now, with the additional information given by Λ, the domination relation is that A_i is better than A_j whenever

$$v(A_i)\lambda \geq v(A_j)\lambda \quad \forall \lambda \in \Lambda, \tag{3}$$

with at least one strict inequality. If we denote by ext(Λ) the set of extreme points of Λ, then (3) is equivalent to

$$v(A_i)\lambda^e \geq v(A_j)\lambda^e \quad \lambda^e \in \text{ ext } (\Lambda), \tag{4}$$

with at least one strict inequality. Therefore, denoting L as the matrix whose columns are the extreme points of Λ, then (4) can be rewritten as

$$v(A_i)L \gneqq v(A_j)L.$$

In other words, in order to handle multi-attribute data with a set of information Λ, we should use the transformed data given by

$$v(A_i, \Lambda) = v(A_i)L. \tag{5}$$

It is worth noting that the extreme points of $\Lambda^{\leq}$ are $(1, 0, \ldots, 0), \ldots, (0, \ldots, 0, 1)$; thus, (1) is just a particular case of the more general case given in (4). It is clear that the domination relation for the alternatives of the decision process evaluated with the transformed data $v(A_i, \Lambda)$, $i = 1, \ldots, n$ is again given componentwise. In this sense, we claim that the additional information has been incorporated to the data.

Hence, the problem is now reduced to calculate the extreme points of Λ. The difficulty of this task depends on the structure of the new set of information. When Λ is defined by certain special relations, it is easy to obtain its extreme points (Carrizosa et al., 1995; Mármol et al., 1998a). In general, the extreme points of sets defined by linear relations can be obtained in a sequential way (Mármol et al., 1998b).

Once you have incorporated the information into the data, any criterion V may be applied to evaluate any set of alternatives. Let $V[A_i, \Lambda]$ denote the value that the criterion V assigns to the alternative A_i, using the information on attributes given by Λ.

In this paper, we analyze three classical decision criteria under this perspective: Laplace, Wald and Hurwicz criteria. In order to make compatible the D-M attitude with the additional information, which may come from diverse sources, the D-M should apply the criteria to the data transformed by (5). Thus, the optimistic or pessimistic interpretation of the attitude of the D-M does make sense on the new data, and the classical criteria can be applied.

We study three criteria applied in multi-attribute decision making, proposed by Milnor (1954) in a different context, whose formulas are:

1. *Laplace criterion*

$$V_{\mathrm{L}}(A_i, \Lambda^{\geq}) = \frac{1}{k}\sum_{j=1}^{k} a_{ij}. \tag{6}$$

2. *Optimistic/pessimistic criterion (Wald)*

$$V_{\mathrm{op/w}}(A_i, \Lambda^{\geq}) = \max / \min{}_{j=1, \ldots, k}(a_{ij}). \tag{7}$$

3. *Hurwicz criterion*

$$V_{\mathrm{H}}(A_i, \Lambda^{\geq}) = \gamma \max{}_{j=1, \ldots, k} a_{ij} + (1-\gamma) \min{}_{j=1, \ldots, k} a_{ij}. \tag{8}$$

In order to illustrate the content consider the following example.

Example 1. A decision maker is given with a set of alternatives A_i, $i \in I$ evaluated with respect to four attributes. In the absence of information, the Laplace (6) criterion gives the values

$$V[A_i, \Lambda^{\geq}] = 1/4(a_{i1} + a_{i2} + a_{i3} + a_{i4}) \quad \text{for any } i \in I.$$

Now, assume that the second attribute is not less important than the fourth. This leads to the following information set

$$\Lambda^1 = \{\lambda \in \Lambda^{\geq} : \lambda_2 \geq \lambda_4\}.$$

Then, using (5), the value given by the Laplace criterion is

$$V[A_i, \Lambda^1] = 1/4(a_{i1} + 3/2a_{i2} + a_{i3} + 1/2a_{i4}).$$

Notice that the criterion changes the evaluation for the same alternative once the new information is considered.

So far, we have considered a two-step process to incorporate additional information to the decision process: (1) transform data; (2) apply the criterion. An alternative way to do that is to generate new criteria which already contain the available information so that these criteria can be applied to the original data set. This is what we want to develop in the next sections.

The paper is organized as follows: in Section 2 we develop the expressions for the transformed criteria when different sets of information are assumed. In Section 3 we show how these expressions can be used to easily obtain weighted ratings in qualitative MCDM. The paper ends with some conclusions.

2. Decision criteria with additional information

Assume that the additional information is given by a closed, polyhedral set $\Lambda \subset \Lambda^{\geq}$ whose extreme points are $\{\lambda^r\}$, $r = 1, \ldots, p$. This set modifies the expressions of the evaluations of each alternative in each attribute. Using (5), the transformed data are:

$$v(A_i, \Lambda) = \left(\sum_{j=1}^{k} \lambda_j^1 a_{ij}, \ \ldots, \ \sum_{j=1}^{k} \lambda_j^p a_{ij} \right).$$

Therefore, any criterion applied to these data is modified accordingly. In particular in what follows, we show formulas for three important classical criteria: Laplace, Wald and Hurwicz.

1. *Laplace criterion*

   This criterion evaluates each alternative by its average value. Thus, it represents an intermediate attitude towards risk. The evaluation derived applying (5) to the Laplace criterion given in (6), once the set of weights Λ is considered, is

$$V_{\mathrm{L}}(A_i, \Lambda) = \frac{1}{p} \sum_{r=1}^{p} \sum_{j=1}^{k} \lambda_j^r a_{ij}. \qquad (9)$$

2. *Optimistic/pessimistic criterion (Wald)*

   From an optimistic/pessimistic attitude towards risk, the D-M will evaluate the alternatives as the maximum/minimum value he/she can achieve with the available information. The modified criteria (7), once Λ is considered, are:

$$V_{\mathrm{op/w}}(A_i, \Lambda) = \max / \min_{r=1, \dots, p} \sum_{j=1}^{k} \lambda_j^r a_{ij}. \tag{10}$$

3. *Hurwicz criterion*
   Applying the information set Λ to (8), we get:

$$V_{\mathrm{H}}(A_i, \Lambda) = \gamma \max_{r=1, \dots, p} \sum_{j=1}^{k} \lambda_j^r a_{ij} + (1-\gamma) \min_{r=1, \dots, p} \sum_{j=1}^{k} \lambda_j^r a_{ij}, \tag{11}$$

   where $0 \leq \gamma \leq 1$ represents an optimism coefficient.

Notice that in the case of no information, the extreme points of $\Lambda^{\geq}$ are the canonical basis and the expressions of the classical criteria [(6)–(8)] are obtained from the above formulae and the corresponding $\Lambda^{\geq}$ set.

In the following, we analyze some important cases when additional information about the importance of the attributes is available. Note the types of information which we consider are generated by weighting factors and that we have assumed a closed, polyhedral structure. This makes this approach not exhaustive. For instance, the pure lexicographic information is not covered since its set of weights Λ is neither open nor closed. Nevertheless, as we will see, important cases can be analyzed.

2.1. Lower bounds on the weighting coefficients

When there exist lower bounds on the weights, the information set is given by

$$\Lambda(\alpha) = \{\lambda \in \Lambda^{\geq} : \lambda_j \geq \alpha_j \geq 0\}.$$

We assume that $\Sigma_{j=1}^{k} \alpha_j < 1$ in order to ensure $\Lambda(\alpha) \neq \oslash$. Let $\beta = 1 - \Sigma_{j=1}^{k} \alpha_j$.

The extreme points of this information set are the columns of the following matrix (see Mármol et al., 1998a):

$$\begin{pmatrix} 1 - \sum_{j \neq 1} \alpha_j & \alpha_1 & \dots & \alpha_1 \\ \alpha_2 & 1 - \sum_{j \neq 2} \alpha_j & \dots & \alpha_2 \\ \vdots & \vdots & \dots & \vdots \\ \alpha_k & \alpha_k & \dots & 1 - \sum_{j \neq k} \alpha_j \end{pmatrix}.$$

This leads us to the following expressions for the classical criteria:

1. *Laplace criterion*

$$V_{\mathrm{L}}(A_i,\ \Lambda(\alpha)) = \sum_{j=1}^{k}\alpha_j a_{ij} + \beta\frac{1}{k}\sum_{j=1}^{k}a_{ij}. \tag{12}$$

2. *Optimistic/pessimistic criterion (Wald)*

$$V_{\mathrm{op/w}}(A_i,\ \Lambda(\alpha)) = \sum_{j=1}^{k}\alpha_j a_{ij} + \beta(\max / \min{}_{j=1,\ \ldots,\ k} a_{ij}). \tag{13}$$

3. *Hurwicz criterion*

$$V_{\mathrm{H}}(A_i,\ \Lambda(\alpha)) = \sum_{j=1}^{k}\alpha_j a_{ij} + V_{\mathrm{H}}(A_i,\ \Lambda^{\geq}). \tag{14}$$

The three criteria that we obtain result from the addition of an extra amount to the classical evaluation of the alternatives. This extra amount can be interpreted as a weighted sum of the evaluation of the alternatives with the α_j weighting factor. Note that the higher the value of β, the more the criterion depends on the classical criterion.

As a particular case, consider $\alpha_j = \alpha$, $j = 1, \ldots, k$, which means that all the weights are over the same lower bound. In this situation, Laplace criterion (12) coincides with the classical Laplace criterion (6) because equal bounds produce the same factor in each evaluation. The expression obtained for the optimistic/pessimistic criterion is

$$V_{\mathrm{op/w}}(A_i,\ \alpha) = k\alpha\left(\sum_{j=1}^{k}a_{ij}/k\right) + (1 - k\alpha)(\max / \min_{j=1,\ \ldots,\ k} a_{ij}).$$

These criteria are convex combinations of the classical criteria of Laplace and optimistic/pessimistic. The optimistic case leads to the Cent-dian criterion (Conde et al., 1994).

2.2. Rank ordered attributes

On many occasions, the information on the attributes is given by a preference intercriteria relation. This type of information can be represented by the set

$$\Lambda_{\mathrm{ord}} = \{\lambda \in \Lambda^{\geq} : \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k\},$$

whose extreme points are the columns of the matrix (see Carrizosa et al., 1995)

$$\begin{pmatrix} 1 & 1/2 & 1/3 & \dots & 1/k \\ 0 & 1/2 & 1/3 & \dots & 1/k \\ 0 & 0 & 1/3 & \dots & 1/k \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1/k \end{pmatrix}.$$

These sets have been used in multi-attribute decision making, as for instance in the Dominant Regime Method of Hinloopen et al. (1983) and the Israels and Keller method (1986). Now, the classical criteria [(6)–(8)], modified according to this information set, are:

1. *Laplace criterion*

$$V_{\mathrm{L}}(A_i,\ \Lambda_{\mathrm{ord}}) = \frac{1}{k}\sum_{r=1}^{k}\left(\sum_{j=1}^{r}\frac{a_{ij}}{r}\right). \tag{15}$$

2. *Optimistic/pessimistic criterion (Wald)*

$$V_{\mathrm{op/w}}(A_i,\ \Lambda_{\mathrm{ord}}) = \ \max/\min_{r=1,\ \dots k}\sum_{j=1}^{r}\frac{a_{ij}}{r}. \tag{16}$$

3. *Hurwicz criterion*

$$V_{\mathrm{H}}(A_i,\ \Lambda_{\mathrm{ord}}) = \gamma \max_{r=1,\ \dots k}\sum_{j=1}^{r}\frac{a_{ij}}{r} + (1-\gamma)\min_{r=1,\ \dots k}\sum_{j=1}^{r}\frac{a_{ij}}{r}. \tag{17}$$

These criteria, which are compatible with the information set Λ_{ord}, are equivalent to the classical criteria applied to the average cumulative sums of the original evaluations of the alternatives.

The expression of Laplace criterion (15) can also be written as

$$V_{\mathrm{L}}(A_i,\ \Lambda_{\mathrm{ord}}) = \frac{1}{k}(a_{i1}(1 + 1/2 + \dots + 1/k) + a_{i2}(1/2 + \dots + 1/k) + \dots + a_{ik}(1/k)).$$

Notice that the coefficient assigned to the value of each attribute is the result of sharing a unit of the coefficient of the original criterion with all those attributes ranked in a better place. This is also true when not all the weights are ordered, as can be seen in the following example.

Example 2. Consider the problem of selecting alternatives evaluated with respect to four attributes. Assume that we know that the first and second attributes are not less important than the fourth one. Hence, the information set is

$$\Lambda^2 = \{\lambda \in \Lambda^{\geq} : \lambda_1 \geq \lambda_4,\ \lambda_2 \geq \lambda_4\}.$$

In this situation the evaluation with Laplace criterion consists of choosing the alternative such that the following value is maximized:

$$V[A_i,\ \Lambda^2] = 1/4(4/3a_{i1} + 4/3a_{i2} + a_{i3} + 1/3a_{i4}).$$

Note that compared with the original Laplace criterion given in (6), the weight given to the fourth attribute has been equally distributed between the first, the second and the fourth attributes in the new criterion.

It is worth noting that the set Λ_{ord} gives rise to a new distribution of utilities among the criteria. Each criterion shares uniformly, its utility (given by its weighting coefficient), among those criteria ranked in better positions. As we will see in the next section the uniformity in the distribution disappears if ratio scale inequalities are considered. Thus, uniformity is inherent to pure ordinal information sets.

2.3. Ratio scale inequalities among attributes

In this situation we assume that we are able to give bounds on the ratio between the weighting coefficients of two attributes. This leads us to the set:

$$\Lambda_{\text{rate}} = \left\{ \lambda \in \Lambda^{\geq} : \lambda_1 \geq \frac{m_1}{m_2}\lambda_2,\ \lambda_2 \geq \frac{m_2}{m_3}\lambda_3,\ \ldots,\ \lambda_k \geq 0 \right\},$$

where $m_i \geq 0$, $i = 1, \ldots, k$. The extreme points of these sets are

$$\lambda^i = \frac{1}{m^i}(m_1,\ m_2,\ \ldots,\ m_i,\ 0,\ \ldots,\ 0), \quad i = 1,\ \ldots,\ k,$$

where $m^i = \Sigma_{j=1}^{i}\ m_j$, $i = 1, \ldots, k$ (see Mármol et al., 1998a).

Under this information set we obtain:

1. *Laplace criterion*

$$V_{\text{L}}(A_i,\ \Lambda_{\text{rate}}) = \frac{1}{k}\sum_{r=1}^{k}\sum_{j=1}^{r}\frac{m_j}{m^r}a_{ij}. \tag{18}$$

2. *Optimistic/pessimistic criterion (Wald)*

$$V_{\text{op/w}}(A_i,\ \Lambda_{\text{rate}}) = \max/\min_{r=1,\ \ldots,\ k}\sum_{j=1}^{r}\frac{m_j}{m^r}a_{ij}. \tag{19}$$

3. *Hurwicz criterion*

$$V_{\text{H}}(A_i,\ \Lambda_{\text{rate}}) = \gamma \max_{r=1,\ \ldots,\ k}\sum_{j=1}^{r}\frac{m_j}{m^r}a_{ij} + (1-\gamma)\min_{r=1,\ \ldots,\ k}\sum_{j=1}^{r}\frac{m_j}{m^r}a_{ij}. \tag{20}$$

Note that the structure of these criteria is similar to the ones given in Section 2.2, but the weighting factors are determined by the bounds on the ratios. When $m_i = 1$, $\forall i = 1, \ldots, k$, the former case is obtained.

Notice that even if all the bounds on the ratios are not stated, the criteria can still be constructed in a similar way.

2.4. Rank ordered attributes with discriminating factors

This case is similar to the case studied in Section 2.2, where lower bounds on the differences between the weighting coefficients are considered. These bounds are usually called in the literature, discriminating factors (see Cook and Kress, 1996). The set of information is then

$$\Lambda_{\text{ord}}(\alpha) = \{\lambda \in \Lambda^{\geq} : \lambda_j - \lambda_{j+1} \geq \alpha_j,\ j = 1,\ \ldots,\ k-1,\ \lambda_k \geq \alpha_k\}.$$

We assume that $\Sigma_{j=1}^{k} j\alpha_j < 1$ in order to ensure $\Lambda_{\text{ord}}(\alpha) \neq \emptyset$. Let $\delta = 1 - \Sigma_{j=1}^{k} j\alpha_j$.

This set of information generalizes the case in Section 2.2 and can be extended, also incorporating ratio scales inequalities (see Section 2.3).

If we denote $\sigma = (\Sigma_{j=1}^{k} \alpha_j,\ \Sigma_{j=2}^{k}\alpha_j,\ \ldots,\ \alpha_{k-1} + \alpha_k,\ \alpha_k)$, the extreme points of $\Lambda_{\text{ord}}(\alpha)$ can be written as

$$\lambda^j = \sigma + \frac{1 - \sum_{i=1}^{k} i\alpha_i}{j}(1,\ 1,\ \ldots,\ 1^j,\ 0,\ \ldots,\ 0).$$

See Section 4 in Carrizosa et al. (1995) or Section 3.2 in Mármol et al. (1998a) for a proof.

The extreme points of $\Lambda_{\text{ord}}(\alpha)$ are the sum of two terms, one associated to the vector of discriminating factors α and the other due to the rank order.

Then, the classical criteria become

1. *Laplace criterion*

$$V_{\text{L}}(A_i,\ \Lambda_{\text{ord}}(\alpha)) = \sum_{j=1}^{k} \sigma_j a_{ij} + \frac{\delta}{k} \sum_{r=1}^{k} \left(\sum_{j=1}^{r} \frac{a_{ij}}{r} \right). \tag{21}$$

which coincides with the case of Λ_{ord} plus the term $\Sigma_{j=1}^{k}\ \sigma_j a_{ij}$.

The expression of the Laplace criterion can also be written as

$$V_{\text{L}}(A_i,\ \Lambda_{\text{ord}}(\alpha)) = \sum_{j=1}^{k} a_{ij} \left(\sigma_j + \frac{\delta}{k} \left(\frac{1}{j} + \frac{1}{j+1} + \cdots + \frac{1}{k} \right) \right).$$

2. *Optimistic/pessimistic criterion (Wald)*

$$V_{\text{op/w}}(A_i,\ \Lambda_{\text{ord}}(\alpha)) = \sum_{j=1}^{k} \sigma_j a_{ij} + \delta\ \ \max/\min_{r=1,\ \ldots,\ k} \sum_{j=1}^{r} \frac{a_{ij}}{r}. \tag{22}$$

3. *Hurwicz criterion*

$$V_{\mathrm{H}}(A_i,\ \Lambda_{\mathrm{ord}}(\alpha)) = \sum_{j=1}^{k}\sigma_j a_{ij} + \gamma\delta \max_{r=1,\ \dots,\ k}\sum_{j=1}^{r}\frac{a_{ij}}{r} + (1-\gamma)\delta \min_{r=1,\ \dots,\ k}\sum_{j=1}^{r}\frac{a_{ij}}{r}. \tag{23}$$

These transformed criteria can be interpreted according to the structure of the information set. Since the set $\Lambda_{\mathrm{ord}}(\alpha)$ is a mixture of bounds (Section 2.1) with rank order between the attributes (Section 2.2), the new criteria are similar to those in Section 2.2 plus a fixed term which comes from the lower bounds. Therefore, the criteria result from adding an extra amount to the evaluation of the alternatives applied to the average cumulative sums.

Notice that for $\alpha_j = \alpha,\ j = 1, \dots, k$, the vector $\sigma = (k\alpha, (k-1)\alpha, \dots, 2\alpha, \alpha)$. Then, since $\Sigma_{j=1}^{k} j\alpha_j < 1$, α must verify $\alpha < [2/k(k+1)] = [1/\binom{k+1}{2}]$ and $\delta = 1 - \alpha\binom{k+1}{2}$.

3. Weighted ratings in qualitative MCDM

In this section, we apply the methodology developed in the previous section to calculate overall ratings of alternatives in the presence of ordinal preference, following the model proposed by Cook and Kress (1996). These authors proposed a model to construct the overall ratings of alternatives in the presence of ordinal preferences, both for the importance of the attributes and the ranking of the alternatives. They consider the situation where each one of n alternatives $\{A_i\}$, $i = 1, \dots, n$ is evaluated qualitatively with respect to k attributes $\{C_j\}$, $j = 1, \dots, k$. Each alternative is assigned a rank position $l \in \{1, \dots, L\}$ on each attribute C_j. v_{jl} is the value or worth of being ranked in l-th place on the j-th attribute. These values are not given and must be derived from the ordinal data. Define the composite index for alternative A_i as

$$R_i = \sum_{j=1}^{k}\sum_{l=1}^{L} d_{jl}(i)\lambda_j v_{jl},$$

where

$$d_{jl}(i) = \begin{cases} 1 & \text{if } A_i \text{ is ranked in the } l\text{-th rank position with respect to } C_j \\ 0 & \text{otherwise} \end{cases}.$$

They assume that there exists a threshold, $\alpha_\lambda > 0$, called the attribute discriminating factor, on the ordinal information of the importance of the attributes. This threshold, together with the ordinal information, leads to relationships among the weights λ_j of the form:

$$\lambda_j - \lambda_{j+1} \geq \alpha_\lambda, \quad j = 1, \dots, k-1, \quad \lambda_k \geq \alpha_\lambda.$$

They apply the same argument among the rank position values v_{jl}, for some threshold $\alpha_{\mathrm{C}} > 0$ called the rank positions discriminating factor which gives the following set of constraints:

$$v_{jl} - v_{jl+1} \geq \alpha_{\mathrm{C}}, \quad l = 1, \dots, L-1, \quad v_{jL} \geq \alpha_{\mathrm{C}}, \quad j = 1, \dots, k.$$

These assumptions can be written using our methodology as

$\lambda \in \Lambda_{\mathrm{ord}}(\alpha_\lambda)$ and $v \in \Lambda_{\mathrm{ord}}(\alpha_{\mathrm{C}})$.

For the sake of readability, we will denote in the following $\Lambda_\lambda = \Lambda_{\mathrm{ord}}(\alpha_\lambda)$, $\Lambda_{\mathrm{C}} = \Lambda_{\mathrm{ord}}(\alpha_{\mathrm{C}})$.

The evaluation R_i is derived by giving each alternative the opportunity to choose the best values λ_j and v_{jl} such that its rating is maximized. Formally, the score given by R_i is computed as:

$$V(A_i, \Lambda_\lambda, \Lambda_{\mathrm{C}}) = \max_{(\lambda_j,\ v_{jl})} \sum_{j=1}^{k} \sum_{l=1}^{L} d_{jl}(i)\sigma_j v_{jl}.$$

These values were obtained by Cook and Kress (1996) solving linear programming problems.

Alternatively, the same evaluation can be obtained using directly the criterion $V_{\mathrm{op}}(A_i, \Lambda_{\mathrm{ord}}(\alpha))$ given by (22) in Section 2.4. Therefore, our methodology can be used as an alternative to the procedure by Cook and Kress (1996). In the following, we describe a two-step procedure to calculate those values:

1. Compute $A(\Lambda_{\mathrm{C}}) = (a_{ij}(\Lambda_{\mathrm{C}}))$ with

$$(a_{ij}(\Lambda_{\mathrm{C}})) = (k - l_{ij} + 1)\alpha_{\mathrm{C}} + \frac{\delta}{l_{ij}}, \tag{24}$$

where l_{ij} is the rank position of alternative A_i with respect to the attribute C_j.

2. Set

$$V(A_i, \Lambda_\lambda, \Lambda_{\mathrm{C}}) = V_{\mathrm{op}}(A_i(\Lambda_{\mathrm{C}}), \Lambda_\lambda), \tag{25}$$

where $A_i(\Lambda_{\mathrm{C}})$ is the i-th row of matrix $A(\Lambda_{\mathrm{C}})$.

Note that in the first step the new matrix $A(\Lambda_{\mathrm{C}})$ has been computed only using the criterion (22) and without solving any linear program. Our criterion also ensures that in each column every entry is chosen maximizing the values of the v_{jl} variables among those values verifying the rank position discriminating constraints. Besides, an interesting interpretation results from our analysis. The two-step procedure (24) and (25) is equivalent to the ratings given by Cook and Kress's method. Both steps consist of incorporating information to the raw data using the Wald transformation, then it allows one to see Cook and Kress's method as an optimistic approach.

A detailed analysis of this procedure shows that different criteria could have been used both in steps 1 and 2. We have found that an optimistic criterion is used in Cook and Kress's approach. This implies a positive attitude towards the risk of the D-M. However, assuming a different risk attitude, the Laplace criterion could also have been applied and an alternative analysis would have been obtained. For this situation, the two-step procedure would be:

1. Compute $A(\Lambda_{\mathrm{C}}) = (a_{ij}(\Lambda_{\mathrm{C}}))$ with

$$(a_{ij}(\Lambda_{\mathrm{C}})) = (k - l_{ij} + 1)\alpha_{\mathrm{C}} + \frac{\delta}{l_{ij}} \sum_{t=l_{ij}}^{k} \frac{1}{t}, \tag{26}$$

where l_{ij} is the rank position of alternative A_i with respect to the attribute C_j.

2. Set

$$V(A_i, \Lambda_\lambda, \Lambda_C) = V_L(A_i(\Lambda_C), \Lambda_\lambda), \tag{27}$$

where $A_i(\Lambda_C)$ is the i-th row of matrix $A(\Lambda_C)$.

It is worth noting that (27) provides a rating of the alternatives of the decision process different from the original of Cook and Kress. Nevertheless, it also takes into account the information about the importance of the attributes and the ranking of the alternatives in the same way that Cook and Kress's model does. The difference reduces the risk attitude that the D-M exhibits in making the decision. In (25) the decision-maker is optimistic (he/she uses the maximum criterion), while in (26) the decision-maker is risk neutral (he/she uses the Laplace criterion).

Finally, it is also interesting to note that the application of our approach permits us to extend Cook and Kress's rating ideas to different kinds of information on attributes and ranking of alternatives. For instance, it is straightforward to apply in situations where ratio scale inequalities among attributes (see Section 2.3) and rank positions between alternatives are provided.

Example 3. Consider the problem faced by a consumer choosing from among five automobiles (denoted by $A_1, \ldots, A_5$) (Cook and Kress, 1996). The choice is to be made on the basis of the assessed performance of the cars on three dimensions, safety (C_1), comfort (C_2), and maneuverability (C_3). This performance is given in the form of an ordinal ranking that can be seen in the following table:

Car	C_1	C_2	C_3
A_1	3	1	5
A_2	1	3	3
A_3	2	4	2
A_4	4	2	2
A_5	5	5	1

Assume that the information provided is $C_1 > C_2 > C_3$, that is, C_1 is more important than C_2 and C_2 more important than C_3. From this ordinal ranking, and assuming $\alpha_C = \alpha_\lambda = \alpha$, we will obtain the evaluations of the alternatives when different criteria are used in steps 1 and 2 of our approach.

3.1. First analysis

In this first analysis, we obtain the ratings of the alternatives using the criterion given in (25). For this, we perform the two-steps procedure described previously in (24) and (25).

1. We compute the matrix of evaluations of alternatives $A(\Lambda_\alpha)$ given in (24):

$$A(\Lambda_\alpha) = \begin{bmatrix} 1/3(1-6\alpha) & 1-10\alpha & 1/5(1-10\alpha) \\ 1-10\alpha & 1/3(1-6\alpha) & 1/3(1-6\alpha) \\ 1/2(1-7\alpha) & 1/4(1-7\alpha) & 1/2(1-7\alpha) \\ 1/4(1-7\alpha) & 1/2(1-7\alpha) & 1/2(1-7\alpha) \\ 1/5(1-10\alpha) & 1/5(1-10\alpha) & 1-10\alpha \end{bmatrix}.$$

2. The overall evaluation of each alternative if (25) is used is:

$$V(A_1) = 2/3 - 34/5\alpha + 8\alpha^2$$

$$V(A_2) = 1 - 12\alpha + 24\alpha^2$$

$$V(A_3) = 1/2 - 9\alpha + 7/2\alpha^2$$

$$V(A_4) = 5/12 - 19/6\alpha + 7/4\alpha^2$$

$$V(A_5) = 7/15 - 82/15\alpha + 8\alpha^2.$$

The ratings of the alternatives are given as a function of the threshold α: the discriminating factor between attributes and between rank positions of the alternatives in Cook and Kress's model.

Notice that alternatively (27) could also have been used after (24) instead of (25). This analysis would have meant to be optimistic concerning the rank position of the alternatives and neutral with respect to the importance of the attributes. In this case, the overall evaluation of each alternative would have been:

$$V(A_1) = 68/135 - 182/45\alpha + 8/3\alpha^2$$

$$V(A_2) = 20/27 - 22/3\alpha + 16/3\alpha^2$$

$$V(A_3) = 31/72 - 223/72\alpha + 7/12\alpha^2$$

$$V(A_4) = 25/72 - 163/72\alpha - 7/6\alpha^2$$

$$V(A_5) = 13/45 - 118/45\alpha - 8/3\alpha^2.$$

3.2. Second analysis

In this analysis, we obtain the ratings of the alternatives using the two-step procedure described previously in (26) and (27).

1. Compute the matrix of evaluations of alternatives $A(\Lambda_C)$ given in (26):

$$A(\Lambda_C) = \begin{bmatrix} 47/300 + 13/20\alpha & 137/30 - 37/20\alpha & 12/300 + 8/20\alpha \\ 137/30 - 37/20\alpha & 47/300 + 13/20\alpha & 47/300 + 13/20\alpha \\ 77/300 + 3/20\alpha & 27/300 + 13/20\alpha & 77/300 + 3/20\alpha \\ 27/300 + 13/20\alpha & 77/300 + 3/20\alpha & 77/300 + 3/20\alpha \\ 12/300 + 8/20\alpha & 12/300 + 8/20\alpha & 137/30 - 37/20\alpha \end{bmatrix}.$$

2. The overall evaluation of each alternative using (27) is:

$$V(A_1) = 613/2700 - 1/90\alpha - 11/12\alpha^2$$

$$V(A_2) = 17/50 - 97/90\alpha + 5/3\alpha^2$$

$$V(A_3) = 142/675 + 7/30\alpha + 1/6\alpha^2$$

$$V(A_4) = 209/1350 + 17/30\alpha - 1/3\alpha^2$$

$$V(A_5) = 233/2700 + 13/45\alpha - 3/4\alpha^2.$$

Alternatively, (25) could have been used and the evaluations would have been:

$$V(A_1) = 23/75 - 61/60\alpha + 9/4\alpha^2$$

$$V(A_2) = 137/30 - 11/4\alpha + 15/2\alpha^2$$

$$V(A_3) = 77/300 - 11/60\alpha + \alpha^2$$

$$V(A_4) = 181/900 + 3/20\alpha + 1/2\alpha^2$$

$$V(A_5) = 161/900 - 23/30\alpha + 9/4\alpha^2.$$

In this second analysis similar interpretations to the ones given in the first analysis are possible.

4. Conclusions

Any available additional information must be incorporated to the decision process. In order to make additional information compatible with Pareto dominance, the original data must be transformed according to the information set. Once this transformation is done any criteria can be applied to the transformed data. In this paper, we have developed decision criteria compatible with several information schemes imposed on the set of attributes of the decision process. Moreover, we apply those criteria to obtain weighted ratings in qualitative MCDM when information on the importance of the attributes and ranking of the alternatives are available.

Acknowledgements

The research of the authors has been partially supported by the Spanish Ministerio de Educación y Cultura Grant No. HA1997/0123 and PB97-0707. The authors would also like to acknowledge the referees for their valuable suggestions.

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